

$$4\operatorname{tg}^2 x + \operatorname{ctg}^2 x + 6\operatorname{tg} x - 3 \operatorname{ctg} x - 8 = 0$$

$$4\operatorname{tg}^2 x + \operatorname{ctg}^2 x + 3(2\operatorname{tg} x - \operatorname{ctg} x) - 8 = 0$$

$$2\operatorname{tg} x - \operatorname{ctg} x = t$$

$$t^2 = 4\operatorname{tg}^2 x - 4\operatorname{tg} x \cdot \operatorname{ctg} x + \operatorname{ctg}^2 x$$

$$t^2 + 4 = 4\operatorname{tg}^2 x + \operatorname{ctg}^2 x$$

$$t^2 + 4 + 3t - 8 = 0$$

$$t^2 + 3t - 4 = 0$$

$$x_1 = -4$$

$$x_2 = 1$$

$$2\operatorname{tg} x - \operatorname{ctg} x = -4$$

$$\operatorname{tg} x = y$$

$$2y - 1/y = -4$$

$$2y^2 - 1 + 4y = 0$$

$$2y^2 + 4y - 1 = 0$$

$$D/4 = 4 + 2 = 6$$

$$x_1 = (2 + \sqrt{6})/2$$

$$x_2 = (2 - \sqrt{6})/2$$

$$\operatorname{tg} x = (2 \pm \sqrt{6})/2$$

$$x = \operatorname{arctg}((2 \pm \sqrt{6})/2) + \pi k$$

$$2y - 1/y = 1$$

$$2y^2 - y - 1 = 0$$

$$D = 1 + 8 = 9$$

$$y_1 = (1 - 3)/4 = -1/2$$

$$y_2 = (1 + 3)/4 = 1$$

$$x = \operatorname{arctg}(-1/2) + \pi k$$

$$x = \pi/4 + \pi k$$

Ответ: $\pi/4 + \pi k; \operatorname{arctg}(-1/2) + \pi k; \operatorname{arctg}((2 \pm \sqrt{6})/2) + \pi k$